

# Baryon Mass Splittings in Chiral Perturbation Theory

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Baryon masses are calculated in chiral perturbation theory at the one-loop- $\mathcal{O}(p^3)$  level in the chiral expansion and to leading order in the heavy baryon expansion. Ultraviolet divergences occur requiring the introduction of counter-terms. Despite this necessity, no knowledge of the counter terms is required to determine the violations to the Gell-Mann Okubo mass relation for the baryon octet or to the decuplet equal mass-spacing rule, as all divergences cancel *exactly* at this order. For the same reason all reference to an arbitrary scale  $\mu$  is absent. Neither of these features continue to higher-powers in the chiral expansion. We also discuss critically the absolute necessity of simultaneously going beyond the leading order heavy baryon expansion, if one goes beyond the one-loop- $\mathcal{O}(p^3)$  level. We point out that these corrections in  $1/M_B$  generate new divergences  $\propto m^4/M_{10}$ . These divergences together with the divergences occurring in one-loop- $\mathcal{O}(p^4)$  graphs of chiral perturbation theory are taken care of by the same set of counter-terms. Because of these unknown counter-terms one cannot predict the baryon mass splittings at the one-loop- $\mathcal{O}(p^4)$  level. We point out another serious problem of going to the one-loop- $\mathcal{O}(p^4)$  level. When the decuplet is off its mass-shell there are additional  $\pi N\Delta$  and  $\pi\Delta\Delta$  interaction terms. These interactions contribute not only to the divergent terms  $\propto (m^4/M_{10})$ , but also to nonanalytic terms such as  $\propto (m^4/M_{10})\ln(m/M_{10})$ . Thus without a knowledge of the coupling constants appearing in these interactions one cannot carry out a consistent one-loop- $\mathcal{O}(p^4)$  level calculation.

## I. INTRODUCTION

While chiral perturbation theory ( $\chi$ pt) has a long history [1], modern applications have been driven by the formulation given by Weinberg in 1979. [2] Using power counting techniques, Weinberg demonstrated that for the most general, non-linear chiral lagrangian in the purely mesonic sector a loop expansion can be systematically developed *even* though such lagrangians are not renormalizable in the traditional sense. Infinities generated by loops involving terms of lower chiral power, a quantity which will be defined shortly, are removed by terms of higher power in the lagrangian. The systematics occur because higher power means higher order in an expansion in terms of derivatives of the pion's field and the pion's mass. Provided one restricts kinematically the application of the theory to scales of the order of the pion's mass,  $m$ , such an expansion has at least the hope of converging. The expansion parameter naturally occurring in this loop expansion is  $(m/2\pi f_\pi)^2$ . Of course the introduction of additional terms in the lagrangian requires additional experimental information in order to fix the residual finite piece of these higher power "counter-terms". The number of independent experimental inputs increases rather rapidly with the loop-expansion. For example, while the most general lowest order chiral lagrangian in the mesonic sector,  $\mathcal{L}_2$ , contains only two terms, there are ten independent terms at next order,  $\mathcal{L}_4$ . Nevertheless, nontrivial predictions follow once these new terms are determined. This program was outlined by Weinberg in [2]; its successful implementation in the mesonic sector through the one loop level was performed by Gasser and Leutwyler in their seminal papers of the mid 1980s [3].

The extension of these methods to the nucleon sec-

tor was first attempted by Gasser, Sainio and Svarc [4]. The inclusion of baryons adds the nontrivial complication that the nucleon mass  $M$  is comparable to that of the typical chiral scale  $\chi \sim 2\pi f_\pi$ . A loop expansion, when calculated with the full nucleon propagator [4], inevitably contains terms proportional to  $M/\chi$  and powers thereof. Clearly one does not hope to form a convergent series with such an expansion parameter. Nevertheless the leading infrared,  $(m^2 \rightarrow 0)$  nonanalytical behavior of the graphs did appear in [4] to be systematically correlated with the loop expansion. The authors of [4] thus conjectured that this pattern would continue to all orders in the loop expansion suggesting that such an expansion, if organized properly, would be useful.

Weinberg [5] introduced the notion of chiral power. A general  $2N$  baryon legged graph is assigned the chiral power  $\nu$  given by the expression

$$\nu = 2 - N + 2L + \sum_i V_i (d_i + \frac{1}{2}n_i - 2), \quad (1)$$

in which  $L$  is the number of loops,  $V_i$  is the number of vertices of type  $i$  characterized by  $d_i$  derivatives or factors of  $m$  and  $n_i$  number of nucleon fields. The systematic expansion required that the nucleon be considered nonrelativistic. To the extent that all relevant momentum are of the order of the pion's mass, this constraint is consistent with the entire program of chiral perturbation theory. Weinberg's scheme validated the conjecture of Ref. [4]. We note that we use Eq. (1) in all further discussions to label the power of any particular graph. Subsequent work of Weinberg [6] and others [7] have focussed on the NN force.

By applying techniques developed for heavy quark physics [8,9] to the baryon sector, Jenkins and Manohar

[10] formalized the nonrelativistic treatment of the nucleon and made systematic counting of chiral power possible. All terms proportional to the nucleon's mass are absent by construction, and the loop expansion in terms of momentum and the pion's mass is realized.

The success of the chiral perturbation theory in the nucleon sector relies on a double expansion: a chiral expansion in  $1/\chi$ , and the heavy baryon expansion in  $1/M_B$ . Among graphs with the same number of  $\pi N$  vertices these two expansions are distinct in terms of the parameters of the QCD lagrangian. The chiral expansion is based on the mass of the light quarks  $m_u, m_d, m_s \rightarrow 0$ , while the heavy baryon expansion can be associated with the limit of large  $N_c$  among these graphs.

The first comprehensive application of chiral perturbation theory to the problem of octet and decuplet baryon masses is due to Jenkins [11]. She examined the question why the two well-known predictions, namely, the Gell-Mann Okubo [12] (GMO) relation,

$$\frac{3}{4} M_\Lambda + \frac{1}{4} M_\Sigma - \frac{1}{2} M_N - \frac{1}{2} M_\Xi = 0, \quad (2)$$

and the Decuplet Equal Spacing Rule [13] (DES),

$$\begin{aligned} (M_{\Sigma^*} - M_\Delta) - (M_{\Xi^*} - M_{\Sigma^*}) = \\ (M_{\Xi^*} - M_{\Sigma^*}) - (M_{\Omega^-} - M_{\Xi^*}) = \\ \frac{1}{2} \{ (M_{\Sigma^*} - M_\Delta) - (M_{\Omega^-} - M_{\Xi^*}) \} \end{aligned} \quad (3)$$

work as well as they do despite apparently large corrections coming from the one-loop- $\mathcal{O}(p^3)$  level. The experimental value of the left-hand side of Eq. (2) is  $6.5 \text{ MeV}$  which is 3% of the average intra-multiplet splitting among the octets. The average experimental value of the mass combinations in Eq. (3) is  $27 \text{ MeV}$  which is 20% of the average intra-multiplet splitting among the decuplets. We remind the reader that the two predictions above are based on the assumption that the flavor symmetry breaking term in the lagrangian transforms like the  $\lambda_8$  member of an octet (which is true for QCD) and that its effect may be derived perturbatively. Jenkins went up to  $\mathcal{O}(p^4)$  level by inserting octet and decuplet sigma terms in the loop diagrams and stressed the importance of these terms in explaining the surprising success of GMO and DES.

In this paper we reexamine the application of chiral perturbation theory to the problem of octet and decuplet baryon masses. We use the heavy baryon formalism of Jenkins and Manohar to include the decuplet field [14], and also many of the useful tables which appear in Ref. [11]. We differ from Jenkins on two points, one major and one minor. We also report a new result concerning  $1/M_B$  corrections to the heavy fermion theory. The three points are listed below.

1. As we will see later, divergences occur at the one-loop- $\mathcal{O}(p^3)$  level when the internal baryon and the external baryon are in different flavor multiplets.

The resulting counter-terms are combinations of flavor singlet and flavor octet (specifically, the  $\lambda_8$  member). Nevertheless one can predict the results of GMO and DES, because the flavor structure of the counter-terms ensures that they do not contribute to these mass combinations. We also note that the counter-terms have structures similar to those appearing in  $\mathcal{L}^0$  and  $\mathcal{L}^1$ , but have higher chiral power, namely,  $\mathcal{O}(p^2)$ . When one goes to the one-loop- $\mathcal{O}(p^4)$  level by inserting octet and decuplet sigma terms one needs two types of counter-terms not present in  $\mathcal{L}^0$  or  $\mathcal{L}^1$ . The wavefunction renormalization counter-terms arising from Fig. 1a are  $\mathcal{O}(p^2)$  flavor octets. They contribute through diagrams containing one of these terms and a sigma term separated by a baryon propagator. The net effect belongs to flavor  $8 \otimes 8 = 1 \oplus 8 \oplus 8 \oplus 10 \oplus \bar{10} \oplus 27$  space and contributes to GMO and DES. The vertex renormalization graphs shown in Fig. 1b generate counter-terms of  $\mathcal{O}(p^3)$  proportional to the square of the quark mass matrix. Hence they also belong to flavor  $1 \oplus 8 \oplus 8 \oplus 10 \oplus \bar{10} \oplus 27$  and will also contribute to GMO and DES. The most important point is that these counter-terms are not of forms already present in  $\mathcal{L}^0$  or  $\mathcal{L}^1$ . Unlike multiplicatively renormalizable theories where one can discuss various regularization schemes (e.g. MS or  $\overline{\text{MS}}$ ) without changing the underlying number of inherent parameters in the theory, these counter-terms have residual finite pieces that require further experimental input to determine. We do not know the relevant coupling constants and, hence, cannot predict the values of the GMO or DES mass combinations. We need experimental values of these combinations and other experimental data to determine the unknown coupling constants. Thus we are forced to conclude that so far as GMO and DES are concerned we have at present power to predict only up to the one-loop- $\mathcal{O}(p^3)$  level and not beyond that.

We calculate the left hand sides of Eqs. (2) and (3) at the one-loop- $\mathcal{O}(p^3)$  level only. In principle, these results are thus contained, at least in part (see point 2 below) in the results of Ref. [11], but not explicitly identified. It is important to know what the results are at one-loop- $\mathcal{O}(p^3)$  level because we find that this is the limit of predictability of chiral perturbation theory in the area of baryon masses.

We note that the GMO results at the one-loop- $\mathcal{O}(p^3)$  level have been published already by Bernard *et al* [15]. Similar results for the DES, contained here, are new.

2. The physical value for the mass difference between the decuplet and octet baryons,  $M_{10} - M_8 \approx 2m$ , and it should share with  $m$  the chiral power 1. Then, according to Eq. (1), the decuplet-octet mass

difference term in the lagrangian has chiral power 0 and is included in  $L_v^0$  of Jenkins [11]. The value of a baryon-meson loop is expressed with the help of a function  $W(m, \delta, \mu)$  parametrized by the meson mass,  $m$ , the renormalization scale,  $\mu$ , and the quantity  $\delta$  defined below:

$$\begin{aligned} \text{Octet} - \text{octet} \quad \delta &= 0, \\ \text{Octet} - \text{decuplet} \quad \delta &= M_{10} - M_8, \\ \text{Decuplet} - \text{octet} \quad \delta &= M_8 - M_{10}, \\ \text{Decuplet} - \text{decuplet} \quad \delta &= 0. \end{aligned} \quad (4)$$

The first label on the left of each line is for the external leg and the next one is for the internal leg. This function, defined by Eqs. (26), (27) and (28), has a branch point at  $\delta = \mp m$ , reflecting the instability of the decuplet (octet) to decay into an octet (decuplet) and a meson when the masses allow the process. Because of the proximity of the branch point and because both GMO and DES involve cancellation among large quantities, we argue that the role of  $\delta$  should not be treated perturbatively [11]. \* It should be included in all orders [17]. In Table 1, which appears later in the paper, where we justify our argument, we show that the difference between the results of perturbative and exact treatments of the  $\delta$  term is of the order of the experimental values of the left hand sides of Eqs. (2) and (3).

We note that interesting questions concerning the two limits,  $\delta \rightarrow 0$  and  $m \rightarrow 0$ , in the context of large  $N_c$  have been discussed by Cohen and Broniowski [18].

3. The leading  $1/M_B$  corrections to the heavy fermion theory results is  $\sim (m^4/M_B)$ . We find that when the internal baryon is a decuplet, the  $1/M_B$  corrections to the one-loop result is actually divergent. Specifically, it has the form  $\sim (m^4/M_{10})(\frac{1}{\epsilon} - \gamma_E + \ln(4\pi))$ . This result has important implications. It means that one must add counter-terms  $\sim (m^4/M_{10})$  to be fixed with the help of experimental data. It is not possible to calculate the  $1/M_B$  correction terms *ab initio*. We have seen earlier that counter-terms  $\sim m^4$  are needed when one goes to the one-loop- $\mathcal{O}(p^4)$  level by inserting sigma terms into one-loop- $\mathcal{O}(p^3)$  graphs. Having the same flavor  $SU(3)$  group structure both counter-terms will be determined together from the same experimental information, namely, the octet and decuplet masses. We cannot separate the contributions to

the experimentally fixed counter-term from the two mechanisms -  $1/M_B$  corrections and sigma term insertions.

We discover an additional complication for future one-loop- $\mathcal{O}(p^4)$  level calculations. The  $\pi N \Delta$  [19] and  $\pi \Delta \Delta$  couplings each contain an additional term which is  $1/M_B$  suppressed compared to the term retained in the heavy fermion theory. These terms contribute to the ultraviolet divergent term in  $\frac{m^4}{M_{10}}$ . This by itself is not a matter of concern. As we have noted above one can only fix the strength of the total  $\sim m^4$  counter-term and not the part coming from  $1/M_B$  effects. But these divergences are also accompanied by finite, nonanalytic terms in  $m/M_B$ . Such terms must thus be calculated and included in the expressions for the baryon masses used to determine the counter-terms. But it cannot be done without knowing the values of the secondary coupling constants. As these coupling constants play their roles only when the  $\Delta$  is off its mass shell, to fix them reliably from experiment in a credible manner may prove to be a nearly impossible task. The point is illustrated by the work of Benmerrouche, Davidson, and Mukhopadhyay [20]. They attempted to fix the secondary coupling constant  $\alpha$ , defined later in Eq. (20), which appear in  $\pi N \Delta$  interaction and were able only to place its value within a rather broad range, namely,  $0.30 \geq \alpha \geq -0.78$ .

The lowest order lagrangian depends upon four coupling constants:  $D$  and  $F$  describe meson-octet couplings,  $\mathcal{C}$  baryon octet-decuplet couplings and,  $\mathcal{H}$  meson-decuplet couplings. The values of the first three are reasonably well determined. The GMO combination of masses, depends only on these quantities and, using the values of Jenkins [11], we obtain typically  $9 \text{ MeV}$  while the experimental value is  $6.5 \text{ MeV}$ . The value of the decuplet spacing depends upon  $\mathcal{H}$  which is difficult to determine experimentally. If we chose to fit the average violation to the decuplet equal spacing rule ( $27 \text{ MeV}$ ) at the one-loop- $\mathcal{O}(p^3)$  level we obtain  $\mathcal{H}^2 = 6.6$ .

## II. BARYON SELF-ENERGIES

### A. The purely Octet sector

Up to  $\mathcal{O}(p^3)$  the effective chiral lagrangian coupling octet pseudoscalar mesons to octet baryons is: [9,11]

$$\begin{aligned} \mathcal{L}_{eff} &= \mathcal{L}_0^{\pi N} + \mathcal{L}_1^{\pi N} + \mathcal{L}_2^{\pi\pi} \\ \mathcal{L}_0^{\pi N} &= \text{Tr} \bar{B} (i \not{D} - M_B) B + \\ &\quad D \text{Tr} \bar{B} \gamma^\mu \gamma_5 \{A_\mu, B\} + F \text{Tr} \bar{B} \gamma^\mu \gamma_5 [A_\mu, B] \\ \mathcal{L}_1^{\pi N} &= b_D \text{Tr} \bar{B} \{ \xi^\dagger M \xi^\dagger + \xi M \xi, B \} \\ &\quad + b_F \text{Tr} \bar{B} [ \xi^\dagger M \xi^\dagger + \xi M \xi, B ] \end{aligned}$$

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\*There are circumstances where such a treatment is appropriate, as in the case of isospin splittings discussed recently by Lebed [16].

$$+\sigma \text{Tr} M(\Sigma + \Sigma^\dagger) \text{Tr} \bar{B} B$$

$$\mathcal{L}_2^\pi = \frac{f_\pi^2}{4} \text{Tr} \partial_\mu \Sigma \partial^\mu \Sigma^\dagger + a \text{Tr} M(\Sigma + \Sigma^\dagger), \quad (5)$$

in which,

$$\xi = e^{i\pi/f_\pi}, \quad \Sigma = \xi^2 = e^{i2\pi/f_\pi},$$

$$V_\mu = \frac{1}{2}[(\partial_\mu \xi)^\dagger \xi + \xi^\dagger (\partial_\mu \xi)], \quad A_\mu = \frac{i}{2}[(\partial_\mu \xi)^\dagger \xi - \xi^\dagger (\partial_\mu \xi)],$$

$$D^\mu B = \partial^\mu B + [V^\mu, B]. \quad (6)$$

The definitions of the mass matrix,  $M$ , and the octet meson and baryon fields are, by now, standard, and are given in Ref. [9,11]. Note that the subscripts on the baryonic sector of  $\mathcal{L}_{eff}$  refer to the chiral power defined by Eq. (1).

The one loop nucleon self-energy,  $\Sigma(p, M_B)$ , is shown diagrammatically in Fig. (2). The expression for  $\Sigma(P, M_B)$  on mass-shell is given by

$$\Sigma(P, M_B) = \frac{i\beta}{2f_\pi^2} \int \frac{d^4 k}{(2\pi)^4} \frac{\gamma_5 \not{k} (\not{P} + \not{k} + M_B) \gamma_5 \not{k}}{(k^2 - m_\pi^2 + i\eta)(2P \cdot k + k^2 + i\eta)}$$

$$= \frac{-i\beta}{2f_\pi^2} \int \frac{d^4 k}{(2\pi)^4} \frac{(M_B + \not{P})k^2}{(k^2 - m_\pi^2 + i\eta)(2P \cdot k + k^2 + i\eta)}, \quad (7)$$

where  $\beta$  represents SU(3) algebra factors.

The heavy baryon result [11] for  $\Sigma(P, M_B)$  can be obtained by introducing  $P = mv$  and taking the  $M_B \rightarrow \infty$  limit of the *integrand* in the above, whereby one obtains that

$$\Sigma(P, M_B \rightarrow \infty) = \frac{-i\beta}{2f_\pi^2} \int \frac{d^4 k}{(2\pi)^4} \frac{\frac{1}{2}(1 + \not{v})k^2}{(k^2 - m_\pi^2 + i\eta)(v \cdot k + i\eta)}. \quad (8)$$

The same result is obtained by first reducing the effective lagrangian  $\mathcal{L}_{eff}$  in the heavy fermion limit in terms of velocity fields  $B_v$ , [9]

$$\frac{1}{2}(1 + \not{v})B(x) = e^{-iM_B v \cdot x} B_v(x). \quad (9)$$

Any reference in  $\mathcal{L}_{eff}$  to  $M_B$  is thereby removed, so that, for example,  $\mathcal{L}_0^N$  becomes [10]

$$\mathcal{L}_v^0 = i \text{Tr} \bar{B}_v v \cdot D B_v + 2D \text{Tr} \bar{B}_v S_v^\mu \{A_\mu, B_v\} + 2F \text{Tr} \bar{B}_v S_v^\mu [A_\mu, B_v]. \quad (10)$$

where  $S_v^\mu$  is a spin factor defined in Refs. [10,14]. Observe that from  $\mathcal{L}_v^0$ , the nucleon's propagator is given directly to be  $i/(v \cdot k + i\eta)$ .

As in previous works, [4,11] we use dimensional regularization to evaluate all integrals. In the purely mesonic sector it is well known that dimensional regularization, by not introducing any additional mass parameters, avoids complications [22] in the path-integral arising from the chiral-invariance of the measure. We know of no such similar result involving the baryon sector but find that

the use of alternative regularization schemes such as Euclidean cutoff, that introduces additional mass parameters, would complicate the power counting result of Weinberg, Eq. (1). In order to avoid these complications we use dimensional regularization.

In Appendix A we present one method of evaluating Eq. (8). One finds that the mass-splitting  $\delta M_B$  is given simply in the heavy baryon limit by

$$\delta M_B \Big|_{M_B \rightarrow \infty} = \beta \frac{-m^3}{16\pi f_\pi^2}. \quad (11)$$

The wavefunction normalization,  $Z_2$ , is given by the following expression.

$$Z_2^{-1} = 1 - \frac{m^2}{8\pi^2 f_\pi^2} \left( \frac{1}{\epsilon} - \gamma_E + 1 + \ln(4\pi) - \ln \frac{m^2}{\mu^2} \right). \quad (12)$$

Clearly  $Z_2$  requires renormalization which is accomplished through counterterms of chiral power 2 in  $\mathcal{L}_{eff}$ . These have been given by Lebed and Luty [23]. These authors have suggested that the wavefunction renormalization counterterms may be absorbed by redefining the baryon field. Although this is correct for the  $\text{Tr} \bar{B} i \not{D} B$  term in the lagrangian, such a redefinition will necessarily generate new interaction terms. For example, otherwise charge conservation, which requires cancellation between wavefunction renormalization and vertex renormalization ( $Z_1 = Z_2$ ), cannot be maintained. Thus the need to deal with these counterterms cannot be avoided. Use of only the logarithmic piece in  $Z_2$  is not sufficient [10,11,14,17,24].

For the present case, by confining ourselves to only the one-loop- $\mathcal{O}(p^3)$  level, we avoid the complications of the wavefunction renormalization as well as the  $1/M_B$  corrections discussed earlier.

We will now discuss the inclusion of the decuplet, which involves its own unique features.

## B. The Decuplet

The decuplet is included as a spin 3/2 Rarita-Schwinger field [25]  $\Delta^\mu$ . On-shell,  $\Delta^\mu$  obeys the Dirac equation

$$(i \not{\partial} - M_{10}) \Delta^\mu = 0 \quad (13)$$

along with the constraints

$$\gamma_\mu \Delta^\mu = 0, \quad \partial_\mu \Delta^\mu = 0, \quad (14)$$

which eliminate the spin 1/2 components of the  $\Delta^\mu$  field. The most general free lagrangian for  $\Delta^\mu$  that generates the Dirac equation of motion and the constraints is [19,26,20,21]

$$\begin{aligned}\mathcal{L}_\Delta = & -\overline{\Delta}^\mu [(i \not{\partial} - M_{10})g_{\mu\nu} + iA(\gamma_\mu \partial_\nu + \gamma_\nu \partial_\mu) \\ & + \frac{1}{2}(3A^2 + 2A + 1)\gamma_\mu \partial^\alpha \gamma_\alpha \gamma_\nu \\ & + M_{10}(3A^2 + 3A + 1)\gamma_\mu \gamma_\nu] \Delta^\nu, \quad (15)\end{aligned}$$

where  $A$  is an arbitrary (real) parameter subject to the one requirement that  $A \neq -1/2$ . Taking  $A = -1$  leads to the most commonly used expression for the decuplet propagator,

$$G^{\mu\nu} = \frac{1}{i} \frac{P + M_{10}}{P^2 - M_{10}^2 + i\eta} \left[ g^{\mu\nu} - \frac{1}{3} \gamma^\mu \gamma^\nu - \frac{2}{3} \frac{P^\mu P^\nu}{M_{10}^2} + \frac{P^\mu \gamma^\nu - P^\nu \gamma^\mu}{3M_{10}} \right]. \quad (16)$$

To leading order in the heavy baryon expansion, where [8,14] one takes  $P = M_8 v + k$ , the decuplet propagator becomes

$$\begin{aligned}G_v^{\mu\nu} = & \frac{1}{i} \frac{\frac{1}{2}(1+v)}{v \cdot k - \delta + i\eta} \left[ g^{\mu\nu} - \frac{1}{3} \gamma^\mu \gamma^\nu - \frac{2}{3} v^\mu v^\nu \right. \\ & \left. + \frac{1}{3} (v^\mu \gamma^\nu - v^\nu \gamma^\mu) \right] \\ \equiv & \frac{1}{i} \frac{\frac{1}{2}(1+v)}{v \cdot k - \delta + i\eta} P_v^{\mu\nu}. \quad (17)\end{aligned}$$

The quantity  $\delta$ , which we take to be  $226 \text{ MeV}$ , is the mass difference between the baryon octet and baryon decuplet masses.

The constraints on the decuplet field in the heavy baryon theory have been given by Jenkins and Manohar [14]

$$\begin{aligned}\gamma_\mu \Delta^\mu &= 0, \\ v_\mu \Delta^\mu &= 0. \quad (18)\end{aligned}$$

The most general, chirally invariant interaction lagrangian involving decuplets, octet baryons and octet mesons is:

$$\begin{aligned}\mathcal{L}^i = & \mathcal{C}(\overline{\Delta}^\mu \Theta_{\mu\nu} A^\nu B + h.c.) + \mathcal{H} \overline{\Delta}^\mu \gamma_\nu \gamma_5 A^\nu \Delta_\mu \\ & + \tilde{H}(\overline{\Delta}^\mu \gamma_\mu \gamma_5 A^\nu \Delta_\nu + h.c.) \quad (19)\end{aligned}$$

where  $\Theta_{\mu\nu}$  is given by [19,20]

$$\Theta_{\mu\nu} = g_{\mu\nu} + \alpha \gamma_\mu \gamma_\nu. \quad (20)$$

In the heavy fermion theory the last term vanishes and the first two terms become the interaction terms [14]:

$$\mathcal{L}_v^i = \mathcal{C}(\overline{\Delta}^\mu A^\mu B + h.c.) + 2\mathcal{H} \overline{\Delta}^\mu S_{\nu\nu} A^\nu \Delta_\mu. \quad (21)$$

Using the decuplet propagator given by Eq. (17) and the constraints given by Eq. (18), one obtains for the mass-shift from Fig. (3)

$$\begin{aligned}\delta M' = & \frac{-i3\beta'}{4f_\pi^2} \int \frac{d^4 k}{(2\pi)^4} \frac{k_\mu k_\nu P_v^{\mu\nu}}{(k^2 - m^2 + i\eta)(v \cdot k - \delta + i\eta)}, \\ = & \frac{\beta'}{16\pi f_\pi^2} \left[ \frac{-3\delta}{2\pi} (m^2 - \frac{2}{3}\delta^2) \left( \frac{1}{\epsilon} - \gamma_E + \ln(4\pi) - \ln \frac{m^2}{\mu^2} \right) \right. \\ & \left. - \frac{3\delta m^2}{2\pi} - \frac{2}{\pi} (m^2 - \delta^2)^{3/2} \tan^{-1} \frac{\sqrt{m^2 - \delta^2}}{\delta} \right]. \quad (22)\end{aligned}$$

It is clear from Eq. (22), and as announced earlier, that upon inclusion of the decuplet, the mass-shift requires renormalization. Two types of counter-terms belonging to  $\mathcal{L}_2^{\pi N}$  are needed, one to cancel the divergence proportional to  $\delta^3$  and the other in  $\delta m^2$ . The  $\delta^3$  term turns into an overall mass-shift when all *relevant* intermediate states are summed over. The  $\delta m^2$  term is a sum of flavor singlet and flavor octet. As noted earlier all counterterms (divergences) cancel *exactly* in the mass combinations which appear in the GMO and the DES. For completeness, the counter-terms are listed in Appendix B.

### III. $1/M_B$ CORRECTIONS

If in subsection II A, instead of adopting the heavy fermion theory, we had evaluated Eq. (7) we would have obtained the following expression for the octet self-energy contribution coming from a loop containing an octet baryon internal line:

$$\begin{aligned}\delta M_B = & \frac{\beta}{16\pi f_\pi^2} \left[ \frac{M_B^3}{\pi} \left( \frac{1}{\epsilon} - \gamma_E + \ln(4\pi) + 1 - \ln M_B^2 \right) \right. \\ & + \frac{M_B m^2}{\pi} \left( \frac{1}{\epsilon} - \gamma_E + \ln(4\pi) + 2 - \ln M_B^2 \right) \\ & \left. - m^3 \left( 1 - \frac{m}{\pi M_B} \left[ 1 + \ln \frac{M_B}{m} \right] \right) + \dots \right]. \quad (23)\end{aligned}$$

The ultraviolet divergences proportional to  $M_B^3$  and  $M_B$  are those first noted by Gasser, Sainio and Svarc [4]. The additional divergent terms obtained by letting  $M_B \rightarrow \infty$  have identical flavor structure. Observe that no nonanalytical behavior in  $m_\pi$  is thus lost in the  $M_B \rightarrow \infty$  limit.

The new information contained in Eq. (23) is the  $1/M_B$  correction term in the last line. The contribution of the correction term to GMO is  $\sim 40\%$  of that of the  $m^3$  term. Thus it is quite substantial. Unfortunately, it is not enough to include finite, chirally nonanalytic terms like these to obtain the leading  $1/M_B$  corrections. The reason is that a one-loop graph containing a decuplet internal line gives a divergent contribution proportional to  $m^4/M_B$ . They arise from the presence of the  $-\frac{2}{3} \frac{P^\mu P^\nu}{M_{10}^2} + \frac{P^\mu \gamma^\nu - P^\nu \gamma^\mu}{3M_{10}}$  terms in the decuplet propagator given by Eq. (16). Within a loop  $P = p + k$ , where  $p$  is the external momentum and  $k$  the loop momentum. The appearance of extra powers of the loop momentum is responsible for the divergent  $1/M_B$  terms.

The interaction terms  $\mathcal{C}\alpha(\overline{\Delta}^\mu \gamma_\mu \gamma_\nu A^\nu B + h.c.)$  and  $\tilde{H}(\overline{\Delta}^\mu \gamma_\mu \gamma_5 A^\nu \Delta_\nu + h.c.)$ , which appear in Eq. (19), also contribute to the divergent terms. The combined results are shown below.

$$\delta M_8 : \frac{-1}{64f_\pi^2 \pi^2} \left[ \frac{(M_8 + M_{10})}{M_{10}} (1 + \alpha)(2 - \alpha) - 3\alpha(1 + \alpha) \right]$$

$$\sum_{\lambda} \beta'_8(\lambda) \frac{m_{\lambda}^4}{M_{10}} \left[ \frac{1}{\epsilon} - \gamma_E + \ln(4\pi) \right]$$

$$\delta M_{10} : - \frac{3[1 - 2\tilde{H}/\mathcal{H} - 3(\tilde{H}/\mathcal{H})^2]}{320f_{\pi}^2\pi^2}$$

$$\sum_{\lambda} \beta_{10}(\lambda) \frac{m_{\lambda}^4}{M_{10}} \left[ \frac{1}{\epsilon} - \gamma_E + \ln(4\pi) \right] \quad (24)$$

The presence of these counter-terms has four important consequences.

First, we can no longer calculate the  $1/M_B$  corrections completely.

Second, the flavor structure of the divergences and, hence, of the counter-terms is identical to those appearing at the one-loop- $\mathcal{O}(p^4)$  level due to the insertion of sigma terms into one-loop- $\mathcal{O}(p^3)$  graphs. The sum of the two groups of counter-terms have to be fixed with the help of experimental data, which may include the octet and decuplet mass splittings themselves.

Third, since the two share the same counter-term, when one goes to the one-loop- $\mathcal{O}(p^4)$  level in chiral perturbation theory one must also include the leading  $1/M_B$  corrections. The finite, nonanalytic terms from both sources must be regarded, *a priori*, as equally important. This leads us to the next consequence.

The fourth consequence may prove to be a serious impediment to any one-loop- $\mathcal{O}(p^4)$  calculation. The  $m^4/M_{10}$  divergent terms are inevitably accompanied by nonanalytic terms, e.g.  $(m/M_{10})^4 \ln(m/M_{10})$  in Eq. (23)). These terms must be calculated and included in expressions used to fix the full  $m^4$  counter-terms. Unfortunately, these log terms depend on the quantities  $\alpha$  and  $\tilde{H}$ . As stated earlier, these coupling constants affect physical results only through the role of virtual  $\Delta$ s. Until these constants can be reliably determined from experiment, we are at an impasse in the application of chiral perturbation theory in the nucleon sector.

Anatomy of the  $1/M_B$  divergences suggests that the pattern will continue to chiral perturbation theory results involving more loops and powers. The  $1/M_B$  correction to the heavy fermion theory result at any level will involve divergent terms of a level with one more chiral powers, coming always from internal decuplet lines. These divergences will always be inseparably entangled with chiral divergences of the same level in heavy fermion theory. Thus the  $1/M_B$  correction terms cannot be calculated without a complete calculation at the same level of chiral power. It must be noted that this requirement is imposed not merely by considerations of consistency but by the appearance of divergences. One is not left with any option in the matter.

#### IV. MASS SPLITTINGS

Most of the algebraic quantities used in this section have appeared in Refs. [11,17,27]. For the reader's con-

venience we include the coefficients  $\alpha_i$  and  $\beta_i$  defined by Jenkins [11]. They appear in Tables III and IV.

Following the style of Ref. [11] we write for the mass,  $M_i$ , of the "ith" baryon through the one-loop order as

$$M_i = M_B + \frac{1}{2}(1 \mp 1)\delta + \alpha_i + \alpha_i^{\delta}(\mu) \frac{3\delta\zeta}{32\pi^2 f_{\pi}^2}$$

$$- \Sigma_{\lambda} \beta_i(\lambda) \frac{m_{\lambda}^3}{16\pi f_{\pi}^2} - \frac{\delta^3}{16\pi^2 f_{\pi}^2} a_{R}^{\pm, \Delta}(\mu)$$

$$- \Sigma_{\lambda} \beta'_i(\lambda) W(m_{\lambda}, \delta, \mu). \quad (25)$$

In the second term the upper (lower) sign is for octet (decuplet) baryons. The coefficients  $\alpha_i$  come from  $\mathcal{L}_1^{\pi N}$ . The term proportional to  $\beta_i$  arises from the chiral loops in Fig. 2 in which the propagating baryon is in the same multiplet as the baryon  $i$ , while the terms proportional to  $\beta'_i$  arises from the loops in Fig. 3 where the propagating baryon comes from the other multiplet. The sum over  $\lambda$  runs over  $\pi$ ,  $K$  and  $\eta$  mesons. The quantities  $\alpha_i^{\delta}$ , are obtained by adding a superscript  $\delta$  to each of the entries in Table III. The set  $\{b_D^{\delta}, b_F^{\delta}, \dots\}$ , thus generated, is defined in the appendix.  $\zeta$  is the proportionality constant in the GMOR [28] relation,  $\zeta = m_K^2/(m_s + \tilde{m}) = m_{\pi}^2/2\tilde{m}$ .

The coefficients  $a_R^{N, \Delta}(\mu)$  and  $\alpha_i^{\delta}(\mu)$  depend implicitly upon a choice of scale,  $\mu$ . This scale appears explicitly in the function  $W(m, \delta, \mu)$  [15,27]. We give below the expressions for the function for three cases of interest:

$$\delta = 0, \quad W(m, \delta, \mu) = \frac{1}{16\pi f_{\pi}^2} m^3, \quad (26)$$

$$m > |\delta|, \quad W(m, \delta, \mu) =$$

$$\frac{1}{8\pi^2 f_{\pi}^2} (m^2 - \delta^2)^{3/2} \tan^{-1} \frac{\sqrt{m^2 - \delta^2}}{\delta}$$

$$- \frac{3\delta}{32\pi^2 f_{\pi}^2} \left( m^2 - \frac{2}{3}\delta^2 \right) \ln \frac{m^2}{4\delta^2}$$

$$- \frac{3\delta}{32\pi^2 f_{\pi}^2} \left( m^2 - \frac{2}{3}\delta^2 \right) \ln \frac{4\delta^2}{\mu^2}, \quad (27)$$

$$|\delta| > m, \quad W(m, \delta, \mu) =$$

$$\frac{-1}{16\pi^2 f_{\pi}^2} (\delta^2 - m^2)^{3/2} \ln \frac{\delta - \sqrt{\delta^2 - m^2}}{\delta + \sqrt{\delta^2 - m^2}}$$

$$- \frac{3\delta}{32\pi^2 f_{\pi}^2} \left( m^2 - \frac{2}{3}\delta^2 \right) \ln \frac{m^2}{4\delta^2}$$

$$- \frac{3\delta}{32\pi^2 f_{\pi}^2} \left( m^2 - \frac{2}{3}\delta^2 \right) \ln \frac{4\delta^2}{\mu^2}, \quad (28)$$

When the SU(3) algebra factors are included the  $W$ 's appear in the combination

$$V(\delta) = \left\{ -\frac{1}{4} W(m_{\pi}, \delta, \mu) + \right.$$

$$\left. W(m_K, \delta, \mu) - \frac{3}{4} W(m_{\eta}, \delta, \mu) \right\} \quad (29)$$

In the present order of the chiral expansion the octet meson mass squares are taken to be proportional to the masses of the current quarks [28]:

$$m_\eta^2 = \frac{4}{3}m_K^2 - \frac{1}{3}m_\pi^2. \quad (30)$$

We note from the Eqs. (26), (27) and (28) that the  $\ln \mu^2$  term appears in these expression with factors which are linear in  $m^2$ . Combining this fact with Eq. (30) it is easy to see that  $V(\delta)$  does not contain terms proportional to  $\ln \mu^2$ .

From Eq. (28) one finds that when  $m \rightarrow 0$  the quantity

$$W(m, \delta, \mu) \simeq \frac{-1}{16\pi^2 f_\pi^2} \left[ \frac{3}{2} \delta (m^2 - \frac{2}{3} \delta^2) \ln \frac{4\delta^2}{\mu^2} + \frac{1}{2} \delta m^2 - \frac{9}{16} \frac{m^4}{\delta} + \frac{3}{8} \frac{m^4}{\delta} \ln \frac{m^2}{4\delta^2} \right]. \quad (31)$$

Thus it is perfectly well-behaved.

Finally one obtains for the Gell–Mann Okubo mass relation the expression

$$\begin{aligned} \frac{3}{4} M_\Lambda + \frac{1}{4} M_\Sigma - \frac{1}{2} M_N - \frac{1}{2} M_{\Xi} = \\ \frac{2}{3} (D^2 - 3F^2) V(0) - \frac{1}{9} \mathcal{C}^2 V(\delta), \end{aligned} \quad (32)$$

and for the violations to the Decuplet Equal Spacing rule:

$$\begin{aligned} (M_{\Sigma^*} - M_\Delta) - (M_{\Xi^*} - M_{\Sigma^*}) = \\ (M_{\Xi^*} - M_{\Sigma^*}) - (M_{\Omega^-} - M_{\Xi^*}) = \\ \frac{1}{2} \{ (M_{\Sigma^*} - M_\Delta) - (M_{\Omega^-} - M_{\Xi^*}) \} = \\ \frac{2}{9} \mathcal{C}^2 V(\delta) - \frac{20}{81} \mathcal{H}^2 V(0). \end{aligned} \quad (33)$$

We remind the reader of our convention, Eq. (4), by which  $\delta$  is a negative quantity in Eq. (33). We also remind the reader that all counterterms have explicitly cancelled in these two relations and that the relations are independent of the scale  $\mu$ .

Before discussing the numerical results following from Eqs. (33) and (32) and their implications, we comment on the accuracy of a perturbative evaluation of the combination  $V(\delta)$ , Eq. (29). The perturbative value is obtained by expanding the combination in a power series of  $\delta$  and retaining only the terms independent of  $\delta$  and linear in  $\delta$ . In Table I we compare GMO and DES using the exact and the perturbative values of  $V(\delta)$  using the parameter set of Ref. [11] given in set 1 of Table II below.

Quantities	Exact	Perturbative
<i>GMO</i> ( <i>MeV</i> )	10.0	-1.1
<i>DES</i> ( <i>MeV</i> )	-4.2	24.2

TABLE I. *GMO* and *DES* using exact and perturbative values of  $V(\delta)$ .

It is clear that the differences are comparable to the experimental values of the mass combinations in the two cases. Hence one cannot treat the effect of  $\delta$  perturbatively.

There are certain difficulties in making numerical prediction at the one loop level. As inputs we need the chiral limit values of the parameters  $D$ ,  $F$ ,  $\mathcal{C}$  and  $\mathcal{H}$ . Consistency requires that these values be extracted from experimental data by using chiral perturbation theory results calculated at the one loop level. As we have discussed earlier, one loop calculations inevitably lead to requiring new and undetermined terms of chiral power 2.

There are serious ambiguities of a different nature involving the coefficients  $\mathcal{C}$  and  $\mathcal{H}$ . The latter can be determined only from the experimental value of the  $\pi\Delta\Delta$  vertex, etc. Needless to say, no such data exists. Hence one must either rely on models or use the results of chiral perturbation theory itself to fix  $\mathcal{H}$ . One such approach has been pursued in Ref. [29], although without having included necessary counter-terms. The quantity  $\mathcal{C}$  can be determined from the decay width of the decuplets. In principle, we should use the value of  $\mathcal{C}$  in the chiral limit. Consistency requires that the decay width be calculated in chiral perturbation theory at the one loop level and compared with the experimental value to extract its chiral limit.

We follow the strategy [14] of extracting  $\mathcal{C}$  using the full (unapproximated) phase-space expression for the decay width

$$\Gamma = \frac{\mathcal{C}^2 \lambda^{3/2} ((M_{10}^2 + M_8^2 - m^2))}{192\pi f_\pi^2 M_{10}^5} \quad (34)$$

where  $\lambda$  is the usual phase-space factor

$$\lambda = M_{10}^4 + M_8^4 + m^4 - 2M_8^2 M_{10}^2 - 2m^2 M_8^2 - 2m^2 M_{10}^2. \quad (35)$$

The average value obtained is  $\mathcal{C}^2 = 2.56$  [14].

One should note an issue which arises from the use of the heavy baryon limit. The latter gives the formula:

$$\Gamma = 2Im\{M_{10}\} = \frac{\mathcal{C}^2 (\delta^2 - m^2)^{3/2}}{12\pi f_\pi^2}. \quad (36)$$

For example, for the case of  $\Delta \rightarrow N + \pi$ , using  $\delta = 292 \text{ MeV}$  and  $\Gamma = 120. \text{ MeV}$ , one obtains from Eq. (36) that  $\mathcal{C}^2 = 1.2$ , while from Eq. (34) that  $\mathcal{C}^2 = 2.2$ . The difference between these two evaluations, nearly a factor of two, arises from what are formally  $1/M_B$  corrections (Eq. (36) is indeed the  $M_B \rightarrow \infty$  limit of (34)). They are nevertheless not small and would have to also be borne in mind when going to higher power.

We present in Table II values of the mass combinations which appear in GMO and Decuplet Equal Spacing (DES) rules. Several sets of parameters have been used for the purpose of comparison.

Quantities	Set 1	Set 2	Set 3	Set 4	Set 5
$D$	0.61	0.56	0.61	0.61	0.61
$F$	0.40	0.35	0.40	0.40	0.40
$\mathcal{C}^2$	2.56	2.56	2.2	1.2	2.56
$\mathcal{H}^2$	3.61	3.61	6.6	3.61	3.61
$m_\pi (MeV)$	140.	140.	140.	140.	0.
$GMO (MeV)$	10.0	8.8	9.0	6.0	12.2
$DES (MeV)$	-4.2	-4.2	27	14.7	-3.9

TABLE II. The value of  $\delta$  is 226 MeV throughout. The experimental value of  $GMO$  is 6.5 MeV and the average value of the violation of  $DES$  is 27 MeV.

The parameters of set 1 are those used by Jenkins [11]. The sets 2, 3 and 4 are designed to show the dependence of the results on the parameters  $D$ ,  $F$ ,  $\mathcal{C}$  and  $\mathcal{H}$ . The set 5 shows the effect of zero pion mass, which comes almost entirely from the change in the mass of  $\eta$  as given by Eq. (30). It is clear that unlike  $GMO$ , the violations to the  $DES$  is particularly sensitive to the parameters  $\mathcal{C}$  and  $\mathcal{H}$ . Since  $\mathcal{H}$  can only be experimentally extracted through loop corrections, the importance of a consistent one-loop evaluation of all the parameters entering the chiral lagrangian must be emphasized. Set 3 contains a typical set of parameters which fit the average violation to the  $DES$  rule.

## V. CONCLUSIONS

We have calculated the combinations of baryon masses, given by Eqs. (2) and (3), which appear in the  $GMO$  and  $DES$  rules, at one-loop- $\mathcal{O}(p^3)$  level in chiral perturbation theory. At this level, these combinations depend neither on any counterterms nor on a renormalization scale  $\mu$ . Some ambiguities remain concerning the values of the coupling constants. We find that one cannot calculate the mass combinations at one-loop- $\mathcal{O}(p^4)$  level because of the presence of undetermined counter-terms required to handle ultraviolet divergences of two varieties. One class of divergences arise from the chiral expansion and involves both wavefunction and vertex renormalization. The other class of divergences arise from the  $1/M_B$  corrections of graphs containing internal decuplet lines. The  $1/M_B$  corrections also give rise to terms which are finite but non-analytic in  $m^2$ . Unfortunately they include a dependence on interactions which arise only when a decuplet is off its mass shell. The associated coupling constants are not known at present.

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## VI. APPENDIX A

It might be illuminating, especially for the issue of  $1/M_B$  corrections, to describe one method of evaluation of the integral in Eq. (8) using dimensional regularization. An alternative approach, with of course the same result, can be found in [11]. Using standard replacements for the nucleon propagator in terms of real and imaginary parts, Eq. (8) can be rewritten as

$$\delta M_B |_{M_B \rightarrow \infty} = \frac{-i\beta}{2f_\pi^2} \int \frac{d^4 k}{(2\pi)^4} \frac{k^2}{(k^2 - m^2 + i\eta)} \times \left( P \frac{1}{v \cdot k} - i\pi \delta(v \cdot k) \right). \quad (37)$$

Note that the integrand arising from the principle valued part of the nucleon's propagator is odd under the transformation  $k \rightarrow -k$  and hence integrates to zero. Working in the nucleon's rest frame, the  $k_0$  integral is used to integrate over the delta-function. Dimensional regularization is then used for the remaining integrals over space-like momenta. We thereby obtain that

$$\begin{aligned} \delta M_B |_{M_B \rightarrow \infty} &= -\frac{\beta}{4f_\pi^2(2\pi)^3} \int d^3 \vec{k} \frac{\vec{k}^2}{\vec{k}^2 + m^2} \\ &= -\frac{\beta}{4f_\pi^2(2\pi)^3} \int d^3 \vec{k} \frac{(\vec{k}^2 + m^2) - m^2}{\vec{k}^2 + m^2} \\ &= \frac{\beta m^2}{4f_\pi^2(2\pi)^3} \int \frac{d^3 \vec{k}}{\vec{k}^2 + m^2} \\ &= \frac{\beta m^2}{32\pi^3 f_\pi^2} \pi^{3/2} \Gamma(-1/2) (m^2)^{1/2} \\ &= -\frac{\beta m^3}{16\pi f_\pi^2}. \end{aligned} \quad (38)$$

The fact that the  $1/M_B$  corrections to this result (given in Eq. (23)) are small might have been anticipated when noting that the singularity at  $v \cdot k = 0$  in (37) is not pinched.

## VII. APPENDIX B

The factor  $a_R^{N,\Delta}(\mu)$  is the residual finite piece of the counterterm in  $\mathcal{L}_2^{\pi N}$  used to renormalize the infinity in Eq. (22) proportional to  $\delta^3$ . Explicitly, these are:

$$\begin{aligned} \mathcal{L}_2^{\pi N} \ni & \frac{\delta^3}{16\pi^2 f_\pi^2} \left( \frac{5}{3} \kappa + a_R^N \right) Tr \bar{B} B \\ & + \frac{\delta^3}{16\pi^2 f_\pi^2} \left( \frac{2}{3} \kappa + a_R^\Delta \right) \bar{\Delta} \Delta \end{aligned} \quad (39)$$

where  $\kappa$  is an ultraviolet divergent constant given by

$$\kappa = \mathcal{C}^2 \left( -\frac{1}{\epsilon} + \gamma_E - \ln(4\pi) \right). \quad (40)$$



The counterterms from  $\mathcal{L}_2^{\pi N}$  that renormalize the infinities in Eq. (22) proportional to  $\delta m^2$  are:

$$\begin{aligned}\mathcal{L}_2^{\pi N} \ni & \frac{3\delta\zeta}{32\pi^2 f_\pi^2} \left( \frac{1}{3}\kappa' + b_D^\delta \right) Tr \bar{B} \{ \xi^\dagger M \xi^\dagger + \xi M \xi, B \} \\ & + \frac{3\delta\zeta}{32\pi^2 f_\pi^2} \left( \frac{-5}{18}\kappa' + b_F^\delta \right) Tr \bar{B} [ \xi^\dagger M \xi^\dagger + \xi M \xi, B ] \\ & + \frac{3\delta\zeta}{32\pi^2 f_\pi^2} \left( \frac{-8}{9}\kappa' + \sigma^\delta \right) Tr M (\Sigma + \Sigma^\dagger) Tr \bar{B} B \\ & + \frac{3\delta\zeta}{32\pi^2 f_\pi^2} (\tilde{m} - m_s) \left( \frac{-1}{27}\kappa' \right) Tr (\Sigma + \Sigma^\dagger) Tr \bar{B} B \\ & + \frac{3\delta\zeta}{32\pi^2 f_\pi^2} \left( \frac{1}{6}\kappa' + c^\delta \right) \bar{\Delta} (\xi^\dagger M \xi^\dagger + \xi M \xi) \Delta \\ & - \frac{3\delta\zeta}{32\pi^2 f_\pi^2} \left( \frac{-1}{6}\kappa' + \tilde{\sigma}^\delta \right) Tr M (\Sigma + \Sigma^\dagger) \bar{\Delta} \Delta \quad (41)\end{aligned}$$

where the ultraviolet divergent constant  $\kappa'$  is given by

$$\kappa' = \mathcal{C}^2 \left( \frac{1}{\epsilon} - \gamma_E + \ln(4\pi) + 1 \right). \quad (42)$$

$\zeta$  is the proportionality constant in the GMOR [28] relation,  $\zeta = m_K^2/(m_s + \tilde{m}) = m_\pi^2/2\tilde{m}$ . The set  $\{b_D^\delta, b_F^\delta, \dots\}$  which enter in the definition of  $\alpha_i^\delta$  in Eq. (25), are the residual finite pieces of these counterterms. Note that the counterterms given in Eqs. (39) and (41) are the only terms from  $\mathcal{L}_2^{\pi, N}$  that contribute to the baryon masses.

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FIG. 1. Contributions to the masses appearing at second order in the chiral counting. The crosses represent an insertion from  $\mathcal{L}_1^{\pi,N}$ . Fig. (a) is the contribution from wavefunction renormalization, Fig. (b) represents a vertex correction; each introduces counterterms needing specification.

FIG. 2. The one-loop self-energy corrections to the baryon in which the intermediate baryon is part of the same multiplet. Dots represent the goldstone mesons; a straight line, the baryon octet; and a double bar, the baryon decuplet.

FIG. 3. The one-loop chiral corrections to the baryon in which the intermediate baryon is not part of the same multiplet. Notation same as in Fig. (1).

$\alpha_N$	$-2b_D m_s - 2\sigma(2\tilde{m} + m_s) - 2b_F(\tilde{m} - m_s)$
$\alpha_\Sigma$	$-2\tilde{m}b_D - 2\sigma(2\tilde{m} + m_s)$
$\alpha_\Xi$	$-2b_D m_s - 2\sigma(2\tilde{m} + m_s) + 2b_F(\tilde{m} - m_s)$
$\alpha_\Lambda$	$\frac{2}{3}(\tilde{m} - 4m_s)b_D - 2\sigma(2\tilde{m} + m_s)$
$\alpha_\Delta$	$2\tilde{m}c - 2(2\tilde{m} + m_s)\tilde{\sigma}$
$\alpha_\Sigma^*$	$(\frac{2}{3}c - 2\tilde{\sigma})(2\tilde{m} + m_s)$
$\alpha_\Xi^*$	$\frac{2}{3}c(\tilde{m} + 2m_s) - 2(2\tilde{m} + m_s)\tilde{\sigma}$
$\alpha_\Omega$	$2cm_s - 2(2\tilde{m} + m_s)\tilde{\sigma}$

TABLE III. The contributions to the baryon masses arising from terms in  $\mathcal{L}_1^{\pi N}$ .  $m_s$  is the mass of the strange quark and  $\tilde{m}$  is the average mass of the up and down quarks.

$$8 \rightarrow 8 \otimes 8$$

$B$	$\sum_{B'}  \langle B' \pi   B \rangle ^2$	$\sum_{B'}  \langle B' K   B \rangle ^2$	$\sum_{B'}  \langle B' \eta   B \rangle ^2$
$N$	$3/2(F + D)^2$	$3F^2 - 2FD + 5/3D^2$	$3/2F^2 - FD + 1/6D^2$
$\Sigma$	$4F^2 + 2/3D^2$	$2F^2 + 2D^2$	$2/3D^2$
$\Lambda$	$2D^2$	$6F^2 + 2/3D^2$	$2/3D^2$
$\Xi$	$3/2(F - D)^2$	$3F^2 + 2FD + 50/3D^2$	$3/2F^2 + FD + 1/6D^2$

$$8 \rightarrow 10 \otimes 8$$

$B$	$\sum_{B^{\frac{3}{2}}}  \langle B^{\frac{3}{2}} \pi   B \rangle ^2$	$\sum_{B^{\frac{3}{2}}}  \langle B^{\frac{3}{2}} K   B \rangle ^2$	$\sum_{B^{\frac{3}{2}}}  \langle B^{\frac{3}{2}} \eta   B \rangle ^2$
$N$	$4/3 \mathcal{C}^2$	$1/3 \mathcal{C}^2$	
$\Sigma$	$2/9 \mathcal{C}^2$	$10/9 \mathcal{C}^2$	$3/9 \mathcal{C}^2$
$\Lambda$	$\mathcal{C}^2$	$2/3 \mathcal{C}^2$	
$\Xi$	$3/9 \mathcal{C}^2$	$\mathcal{C}^2$	$3/9 \mathcal{C}^2$

$$10 \rightarrow 8 \otimes 8$$

$B$	$\sum_B  \langle B \pi   B^{\frac{3}{2}} \rangle ^2$	$\sum_B  \langle B K   B^{\frac{3}{2}} \rangle ^2$	$\sum_B  \langle B \eta   B^{\frac{3}{2}} \rangle ^2$
$\Delta$	$1/3 \mathcal{C}^2$	$1/3 \mathcal{C}^2$	
$\Sigma$	$5/18 \mathcal{C}^2$	$4/18 \mathcal{C}^2$	$3/18 \mathcal{C}^2$
$\Xi$	$1/6 \mathcal{C}^2$	$1/3 \mathcal{C}^2$	$1/6 \mathcal{C}^2$
$\Omega$		$2/3 \mathcal{C}^2$	

$$10 \rightarrow 10 \otimes 8$$

$B$	$\sum_{B^{\frac{3}{2}}}  \langle B^{\frac{3}{2}} \pi   B^{\frac{3}{2}} \rangle ^2$	$\sum_{B^{\frac{3}{2}}}  \langle B^{\frac{3}{2}} K   B^{\frac{3}{2}} \rangle ^2$	$\sum_{B^{\frac{3}{2}}}  \langle B^{\frac{3}{2}} \eta   B^{\frac{3}{2}} \rangle ^2$
$\Delta$	$25/54 \mathcal{H}^2$	$5/27 \mathcal{H}^2$	$5/54 \mathcal{H}^2$
$\Sigma$	$20/81 \mathcal{H}^2$	$40/81 \mathcal{H}^2$	
$\Xi$	$5/54 \mathcal{H}^2$	$30/54 \mathcal{H}^2$	$5/54 \mathcal{H}^2$
$\Omega$		$20/54 \mathcal{H}^2$	$20/54 \mathcal{H}^2$

TABLE IV. The coefficients  $\beta$  and  $\beta'$  for the one-loop contributions to the baryon masses.

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